# **Superluminal pulse reflection and transmission in a slab system doped with dispersive materials**

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(Received 8 March 2004; revised manuscript received 18 June 2004; published 2 December 2004)

The reflection and transmission of a pulse through a slab which is doped with two-level or three-level atoms are investigated theoretically. The doped atoms can be passive (absorptive) or active (gain). We find that both the reflected and transmitted pulses can be superluminal simultaneously for the slab doped with absorptive two-level atoms at the slab thickness equal to  $(2m+1)\lambda_0/4\sqrt{\epsilon_b}$  (where  $\lambda_0$  is the center wavelength of the incident pulse, and  $\epsilon_b$  is the background dielectric constant of the slab) or with active three-level atoms at any thickness. By adjusting the thickness or background dielectric constant of the slab, the reflected pulse can be controlled from superluminal to subluminal or vice versa for the slab doped with absorptive two-level or absorptive three-level atoms. The energy percentage in the reflected pulse can also be controlled by changing the thickness of the slab, and the doped atoms.

DOI: 10.1103/PhysRevE.70.066602 PACS number(s): 42.25.Bs, 42.25.Gy, 42.70.Qs, 41.20.Jb

# **I. INTRODUCTION**

The propagation of a light pulse passing through a dispersive medium has been extensively investigated [1,2]. Superluminal phenomenon (the group velocity is larger than *c* or even becomes negative) has been experimentally observed in absorptive media [3], and pulse tunneling through onedimensional photonic band gaps (1D PBGs) [4–8], and recently in a doublet gain anomalous system [9]. In these investigations, the main attention was focused on the transmission of a pulse through the media. Recently superluminality of the pulse reflection was discussed theoretically in optical phase-conjugating mirror [10] and asymmetric 1D PBGs [11]. In experiments, Longhi *et al.* [12,13] first observed superluminal reflection of an optical pulse by using a double-Lorentzian fiber Bragg grating. Later, Nimtz *et al.* [14] experimentally verified that the reflection delay was almost independent of the barrier's length (this effect is known as the Hartman effect [15]). In these discussions, the reflected pulse is superluminal while the transmitted pulse is subluminal. Blaauboer *et al.* first predicted that superluminal phenomenon may occur simultaneously both in reflection and in transmission by using optical phase conjugation in the unstable regime [10].

In this paper, we consider a pulse incident on a slab system doped with two-level or three-level atoms. The atoms can be passive (absorptive) or active (gain). Some literature has paid attention to the slab systems [16–20], for example, such as a Fabry-Pérot cavities containing gaseous atoms [19], semiconductor cavities with quantum wells [20] and a low-finesse Fabry-Pérot cavity (with silver mirrors) containing absorbing atoms [18]. Reference [18] used the well known effect of vacuum field Rabi splitting [19,21] (i.e., a splitting of the resonance into two resonances) is a result of the strong-coupling interaction between atom and cavity. Using this effect, Manga *et al.* [18] recently showed a way to control the pulse propagation in a cavity with silver mirrors containing resonant absorbing atoms. In fact, in their paper the cavity mode is modified by the resonant absorbing atoms, therefore the transmitted pulse propagation can be controlled from subluminal to superluminal by adjusting atomic density. Agarwal *et al.* [17] also gave out a reciprocity theorem for the reflected and transmitted fields in a slab system. For a lossless system, due to  $|r|^2 + |t|^2 = 1$ , if there is a dip in transmissivity, according to the Kramers-Kronig relations this dip corresponds to an anomalous dispersive region on the curve of the effective refractive index versus frequency [22], which lead to superluminal pulse transmission compared to that of the pulse through the same distance vacuum; for the reflection, if there is a peak in the reflectivity, the phase time of the reflected pulse is positive (i.e., the subluminal pulse reflection). Thus the reflected and transmitted pulses cannot superluminal simultaneously. In our paper, we extend the results of the previous researches [3,9,11,18], and find the reflected pulse can be tuned from subluminal to superluminal simply by controlling the slab's thickness (or the slab's background dielectric constant). Another key point is that we achieve the superluminal pulse reflection and the superluminal pulse transmission simultaneously.

In the present paper, we calculate the peak arrival times of the reflected and transmitted pulses, from which the group velocity can be obtained. Our calculation is under the condition of the narrow spectral limit [7,23–26], and consequently the distortion of the pulse shape is extremely small and can be negligible. Our numerical results show that, it is achievable that both the reflected and the transmitted pulses are superluminal for the doped slab. In the following section, we will present the theory of pulse propagation in a slab system by using the transfer matrix method. In Sec. III, we will discuss the numerical results and consider the pulse reflection and transmission in two kinds of the slabs doped with two-level atoms or three-level atoms, and also give out the role of the slab itself. The final conclusion can be found in Sec. IV.

# **II. PULSE PROPAGATION IN A SLAB**

As we know, a light pulse  $E(z, t)$  can be decomposed into an integral of its Fourier components  $E(z, \omega)$  given by  $E(z,t) = \int E(z,\omega)e^{-i\omega t} d\omega$ . We suppose a pulse be normally propagating into a slab in *z* direction (the slab is in *x*−*y* plane). The extension of the slab is infinite in *x*−*y* plane and is finite from  $z=0$  to  $z=d$  in *z* direction, and the outside of the slab is vacuum. For TE plane-wave pulses (TM planewave pulses can also be discussed in the same way), the transfer matrix for the electric and magnetic components of a monochromatic wave of frequency  $\omega$  through the slab is [24,27]

$$
\left(\begin{array}{c}\n\cos\left[\frac{\omega}{c}n(\omega)d\right] & \frac{1}{n(\omega)}\sin\left[\frac{\omega}{c}n(\omega)d\right] \\
-n(\omega)\sin\left[\frac{\omega}{c}n(\omega)d\right] & \cos\left[\frac{\omega}{c}n(\omega)d\right]\n\end{array}\right),\n\tag{1}
$$

where *c* is the light speed in vacuum and  $n(\omega) = \sqrt{\epsilon(\omega)}$  is the refractive index of the slab with  $\epsilon(\omega)$  the dielectric function. We have assumed the material of the slab is nonmagnetic. Then from the transfer matrix method [24], we can obtain the reflection coefficient  $r(\omega)$  of the monochromatic wave as the following:

$$
i[n(\omega) - 1/n(\omega)]\sin\begin{bmatrix}\omega\\-\frac{1}{n(\omega)}d\\c\end{bmatrix}
$$

$$
r(\omega) = \frac{2 \cos\begin{bmatrix}\omega\\-n(\omega)d\end{bmatrix} - i[n(\omega) + 1/n(\omega)]\sin\begin{bmatrix}\omega\\-n(\omega)d\end{bmatrix}}{c},
$$
(2)

and its transmission coefficient  $t(\omega)$  is given by

$$
t(\omega) = \frac{2}{2 \cos \left[\frac{\omega}{n(\omega)d}\right] - i[n(\omega) + 1/n(\omega)]\sin \left[\frac{\omega}{n(\omega)d}\right]}.
$$
\n(3)

Using the relations between the reflected (or transmitted) field and the input field, i.e.,  $E_r(0, \omega) = r(\omega)E_i(0, \omega)$  and  $E_t(d, \omega) = t(\omega)E_i(0, \omega)$ , where the subscripts "*r*," "*t*," and "*i*" denote the reflected field, the transmitted field and the incident field, respectively, we can obtain the reflected and transmitted pulses given by

$$
E_r(0,t) = \int E_r(0,\omega)e^{-i\omega t}d\omega,
$$
 (4)

$$
E_t(d,t) = \int E_t(d,\omega)e^{-i\omega t}d\omega,
$$
\n(5)

respectively. Thus, we can obtain the result pulses from Eqs. (4) and (5). In the present paper, the spectrum of the incident pulse, shown by the dashed curve in Fig. 1(a), is chosen sufficiently narrow so that the pulse distortion is very small and negligible in our calculation of the peak arriving time. We also assume that the incident pulse is an analytic wave form such as a Gaussian pulse for example. In the limit of



FIG. 1. For the slabs doped with the absorptive two-level atoms, reflectivity and transmittivity versus frequency under different slab's thicknesses (a)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (c)  $d=(2m+1)$  $\langle \times (\lambda_0/4\sqrt{\epsilon_b})$ , respectively. In (a) and (c), the dashed line represents the spectrum of the incident pulse. Figures (b) and (d) show the reflected and transmitted pulses in time domain under the condition (a) and (c), respectively. In (b) and (d), the gray line is the input pulse, and the dashed line is the transmitted pulse, and the black solid line denotes the reflected pulse. All the pulse shapes have been normalized. The other parameters of the slabs are *M* =−10 Hz and  $\gamma$ =0.5 MHz, *m*=1.2×10<sup>5</sup>, and  $\lambda_0$  is corresponding to  $\omega_0$ .

the narrow spectral pulse [24,25], the group delay time can be defined from the peak times  $T_{r,t}^{peak}$  of the transmitted and reflected pulse (i.e., the subscript *r* denotes the reflected pulse and the subscript *t* denotes the transmitted pulse), and the peak time  $T_{r,t}^{peak}$  are equivalent to the phase time delay defined as  $\tau_{r,t} = \partial \phi_{r,t} / \partial \omega$  [1,28,29] (where  $\phi_{r,t}$  are the phases of the reflection and transmission coefficients,  $r(\omega)$  and  $t(\omega)$ , respectively [i.e.,  $r(\omega) = |r(\omega)| \exp[i\phi_r(\omega)]$  and  $t(\omega)$  $=$   $|t(\omega)| \exp[i\phi_t(\omega)]$  ]). From the shapes of the reflected and transmitted pulse, we can obtain the peak times  $T_{r,t}^{peak}$  of the result pulses.

Here we consider the behaviors of both the reflected and transmitted pulses from the slab. For the reflected pulse,  $T_r^{peak}$  < 0 means the superluminal pulse reflection; for the transmitted pulse,  $T_t^{peak} < d/c$  means the superluminal pulse transmission.

### **III. NUMERICAL RESULT AND DISCUSSION**

Let us consider that the incident pulse is a Gaussian pulse at the surface of the slab in the plane  $z=0$ . The electric field of the Gaussian pulse at the incident surface is expressed as

$$
E_i(0,t) = A_0 \exp\left[-\frac{t^2}{2\tau_0^2}\right] \exp[-i\omega_0 t] \tag{6}
$$

and its Fourier spectrum is given by  $E_i(0, \omega)$ = $(\tau_0 A_0 / 2\sqrt{\pi}) \exp[-\tau_0^2 (\omega - \omega_0)^2 / 2]$ . Here  $\tau_0$  is the temporal width of the Gaussian pulse, and  $\omega_0$  is the center frequency. In the following numerical calculations, we take  $\omega_0=2\pi$  $\times 10^{14}$  Hz, and pulse temporal width  $\tau_0=20 \mu s$ .

The slab to be considered here is composed of a constant dielectric material doped by two-level or three-level atoms,

and then the dielectric function  $\epsilon(\omega)$  can be divided into two parts and written as  $\epsilon(\omega)=\epsilon_b+\chi(\omega)$ , where  $\epsilon_b$  is the background dielectric constant (we take  $\epsilon_b$ =4.0 in all our calculations) and  $\chi(\omega)$  is the susceptibility produced by the doped atoms. In the following, we will discuss the propagation of the pulse reflection and transmission from the slabs doped with two different atoms.

#### **A. A slab with two-level atoms**

First we consider the slab is doped by two-level atoms. The linear susceptibility produced by the two-level atoms can be written as

$$
\chi(\omega) = \frac{M}{\omega - \omega_0 + i\gamma},\tag{7}
$$

where *M* is proportional to the oscillator strength depending on the transition dipole moment and the density of the twolevel atoms  $(M>0$  means the gain two-level atoms,  $M<0$ means the absorptive two-level atoms [3,28]), and  $\gamma$  is the phenomenological linewidth.

For an absorptive medium  $(M<sub>0</sub>)$ , there is always a dip in the curve of the transmission versus frequency for any slab length due to the absorption of the medium, and consequently the propagation of the transmitted pulse, whose spectrum is within the absorptive resonant region, is always superluminal [28]. However, the curve of the reflectivity versus frequency is very sensitively dependent on the thickness of the slab. In Figs.  $1(a)$  and  $1(c)$ , we show that the spectral reflectivity and transmittivity for the slab doped with the absorptive two-level atoms  $(M<sub>0</sub>)$  under different thicknesses (i)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (ii)  $d=(2m+1)(\lambda_0/4\sqrt{\epsilon_b})$ ; here *m* is a integer number. The curves of the transmittivity always have a dip near the center frequency  $\omega_0$  in these two cases, so that the propagations of the transmitted pulses in these two case are always superluminal [see the dashed lines in Figs. 1(b) and 1(d)]. Because the effect of the slab itself only slightly change the transmission coefficient, the peak times of the transmitted pulses in these two case are, respectively, about −688 ns and −540 ns (much earlier than the peak time of the incident pulse). But the reflectivity curves for these two cases are completely different from each other. From Fig. 1, we find that, for the case (i), there is a peak in the spectral reflective curve; while for the case (ii), there is a dip. Therefore, we have a subluminal pulse reflection in the case (i), and a superluminal pulse reflection in the case (ii). A numerical example is shown in Figs. 1(b) and 1(d) (see the black solid line). In the case (i), the peak time of the reflected pulse is about  $+1353$  ns (subluminal); in the case (ii), the peak time of the reflected pulse is about −360 ns (superluminal). So there is a transition from the subluminal to the superluminal for the reflected pulse by changing the slab's thickness. The energy in the reflected pulse over the energy of the incident pulse is increased from 2.7% in the case (i) to 23.3% in the case (ii). Therefore, in the case (ii) it gives more energy into the reflected pulse. Here we would like to point out that, we find that *both the transmitted and reflected pulses can be superluminal*, which is different from the previous investigations [4,5,7,8,11,14] in a one-dimensional lossless system. For example, in passive one-dimensional photonic crystals used for the optical tunneling experiments [4,5,7,8], the transmitted pulse can be superluminal (the time delay is positive but much shorter than that of light passing through the same distance vacuum) and at the same time the reflected pulse is subluminal (because the phase time of the reflected pulse is also positive). In asymmetric 1D PBGs [11–13], the reflected pulse is superluminal while the transmitted pulse is subluminal. Here we can obtain both the transmitted and reflected pulses to be superluminal [see Figs. 1(c) and 1(d)]. By adjusting the thickness of the slab, we can also control the behavior of the reflected pulse from subluminal to superluminal (or from superluminal to subluminal). In Ref. [18], the coupling interaction between the cavity's resonance and the atomic resonant absorption is so strong that the transmission resonance is splitted into two resonances by increasing the atomic density (i.e., increasing the absolute value of *M*). In their cases, although they control the transmitted pulse from subluminal to superluminal and control the reflected pulse from superluminal to subluminal, they have not obtained both the superluminal pulse reflection and superluminal pulse transmission simultaneously. In our case, due to that we include the case of the off-resonance effect of the slab when  $d = (2m+1)(\lambda_0/4\sqrt{\epsilon_b})$ , the reflectivity of the doped slab has a dramatic change. This leads to a transition from subluminal to superluminal for the reflected pulse.

In Fig. 2, we plot the reflectivity (a) and transmittivity (b) versus frequency and the thickness. From Fig. 2(a), we can find that there is a peak in the spectral reflectivity when the optical thickness  $L = \sqrt{\epsilon_h}d$  of the slab is equal to an even number of  $\lambda_0 / 4$  and there is always a dip when the optical thickness *L* is an odd number of  $\lambda_0/4$ . Therefore we can change the pulse reflection from subluminal to superluminal or vice versus by adjusting the thickness *d* of the slab (which is also equivalent to changing the background dielectric constant  $\epsilon_b$ ). While for the transmittivity of the doped slab, there is always a dip due to the absorption of the medium [see Fig. 2(b)]. Therefore, it must be superluminal pulse transmission. In Ref. [18], the authors changed the atomic density to make the resonance of the cavity splitting into two resonances, then control the pulse propagation. We do not need to change the density of the doped atoms, and only need to change the thickness *d* or the background dielectric constant  $\epsilon_b$  to control the pulse propagation.

Now let's turn to consider the cases of the slab doped with the gain two-level atoms  $(M>0)$  under different thicknesses (i)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (ii)  $d=(2m+1)(\lambda_0/4\sqrt{\epsilon_b})$ . Due to the gain medium  $(M>0)$ , there is a peak in both the reflection and transmission coefficients [see Figs. 3(a) and  $3(c)$ ], the reflected and transmitted pulses are both subluminal [see Figs. 3(b) and 3(c)]. The peak times are  $+2967$  ns and +924 ns in the case (i) for the reflected and transmitted pulses, respectively. The peak times are +584 ns and +403 ns in the case (ii) for the reflected and transmitted pulses, respectively. The peak delay time for the reflected pulse in case (i) is much longer than that in case (ii). The energy of the reflected pulse over the incident energy is increased from 13.4% in case (i) to 62.7% in case (ii). Therefore, as the



FIG. 2. For the slabs doped with the absorptive two-level atoms, (a) reflectivity and (b) transmittivity as a function of frequency under different slab's thickness. Here  $D=\sqrt{\epsilon_b(d-d_0)}$  and  $d_0=3$  cm. The other parameters are the same with Fig. 1.

thickness of the slab is changed, we can easily increase the reflectivity and obtain much more energy of the reflected pulse than that in case (i) [see Figs. 3(a) and 3(c)]. In both these case, we can obtain the subluminal pulse reflection and the subluminal pulse transmission simultaneously and control the reflected and transmitted behaviors of a pulse.

#### **B. A slab with three-level atoms**

Now we consider the reflection and transmission of a pulse in the slab doped by three-level atoms. The susceptibility of the three-level atoms is a form of double Lorentz oscillators,

$$
\chi(\omega) = \frac{M_1}{\omega - \omega_0 - \Delta + i\gamma} + \frac{M_2}{\omega - \omega_0 + \Delta + i\gamma},
$$
(8)

where  $\Delta$  is the frequency detunning, and  $M_1$  and  $M_2$  are, respectively, proportional to the strength of the two oscilla-



FIG. 3. For the slabs doped with the gain (inverted) two-level atoms, reflectivity and transmittivity versus frequency under different slab's thicknesses (a)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (c)  $d=(2m+1)$  $\times (\lambda_0/4\sqrt{\epsilon_b})$ , respectively. In (a) and (c), the dashed line represents the spectrum of the incident pulse. Figures (b) and (d) show the reflected and transmitted pulses in time domain under the condition (a) and (c), respectively. In (b) and (d), the gray line is the input pulse, and the dashed line is the transmitted pulse, and the black solid line denotes the reflected pulse. All the pulse shapes have been normalized. The other parameters of the slabs are *M* =−10 Hz and  $\gamma = 0.5$  MHz,  $m = 1.2 \times 10^5$ .

tors. It describes a three-level system with two closely placed Raman gain peaks [9,30]. Here we assume  $M_1 = M_2 = M$  for simplicity and consider the two situations with  $M < 0$  for the absorptive slab and  $M > 0$  for the gain slab. For the gain slab system, our study is the extension of the previous investigations [9,30] with the two boundaries (which form a finite cavity in *z* direction) being now taken into account. From the above discussion, we know that there must be some new properties that have not been seen in the previous investigations.

In Fig. 4, we show the cases for  $M < 0$  under the two different thicknesses (i)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (ii)  $d=(2m\sqrt{\epsilon_b})$  $+1$ )( $\lambda_0$ /4 $\sqrt{\epsilon_b}$ ). As shown in Fig. 4(a), the frequency region near the center frequency  $\omega_0$  is normal dispersive, and then the transmitted pulse is subluminal. The peak time of the transmitted pulse is about  $+184$  ns [see Fig. 4(b)]. In Fig. 4(a), it is found that the spectral reflective curve has a dip at the frequency region near the center frequency  $\omega_0$ . Therefore, in this case, the reflected pulse is superluminal with the peak advancement of  $T_r^{peak}$  = −857.7 ns [see Fig. <u>4(</u>b)]. But if we change the thickness of the slab by  $\lambda_0 / 4 \sqrt{\epsilon_b}$  [for case (ii)], the reflective curve is completely different from case  $(i)$ . In this case  $(ii)$ , see Fig.  $4(c)$ , we find that both the transmittivity and reflectivity are normal dispersive around the center frequency, thus both the transmitted and reflected pulses are subluminal with the peak times of  $T_t^{peak}$  $= +132.1$  ns and  $T_r^{peak} = +108.8$  ns, respectively [see Fig. 4(d)]. Therefore, the propagation of the reflected pulse can be controlled from the superluminal to subluminal by adjusting the thickness *d* of the slab. In addition, the energy percentage of the reflected pulse over the incident pulse energy is increased from 0.87% for the case (i) to 28.8% for the case (ii).



FIG. 4. For the slabs doped with the absorptive three-level atoms, reflectivity and transmittivity versus frequency under different slab's thicknesses (a)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (c)  $d=(2m+1)$  $\times(\lambda_0/4\sqrt{\epsilon_b})$ , respectively. In (a) and (c), the dashed line represents the spectrum of the incident pulse. Figures (b) and (d) show the reflected and transmitted pulses in time domain under the condition (a) and (c), respectively. In (b) and (d), the gray line is the input pulse, and the dashed line is the transmitted pulse, and the black solid line denotes the reflected pulse. All the pulse shapes have been normalized. The other parameters of the slabs are  $M = -10$  Hz,  $\Delta$ =0.9 MHz and  $\gamma$ =0.5 MHz,  $m$ =1.2×10<sup>5</sup>.

In Fig. 5, we plot the cases for  $M > 0$  under the different thicknesses (i)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (ii)  $d=(2m+1)$  $\times (\lambda_0/4\sqrt{\epsilon_b})$ . From these figures, we can find that these two cases are similar except that the background reflectivity and background transmittivity far away from the center frequency are very different from each other. The propagations of the pulse reflection and transmission both are superluminal in these two cases. The shapes of the reflected and trans-



FIG. 5. For the slabs doped with the gain (inverted) three-level atoms, reflectivity and transmittivity versus frequency under different slab's thicknesses (a)  $d=2m(\lambda_0/4\sqrt{\epsilon_b})$  and (c)  $d=(2m+1)$  $\times (\lambda_0/4\sqrt{\epsilon_b})$ , respectively. In (a) and (c), the dashed line represents the spectrum of the incident pulse. Figures (b) and (d) show the reflected and transmitted pulses in time domain under the condition (a) and (c), respectively. In (b) and (d), the gray line is the input pulse, and the dashed line is the transmitted pulse, and the black solid line denotes the reflected pulse. All the pulse shapes have been normalized. The other parameters of the slabs are  $M=10$  Hz,  $\Delta$ =0.9 MHz and  $\gamma$ =0.5 MHz,  $m$ =1.2 × 10<sup>5</sup>.



FIG. 6. The reflectivity versus frequency and thickness for the slab without the doped atoms. Here  $D=\sqrt{\epsilon_b(d-d_0)}$  and  $d_0=3.0$  cm.

mitted pulses are also shown in Figs. 5(b) and 5(d). In case (i),  $T_t^{peak} = -210.2$  ns and  $T_t^{peak} = -1249.0$  ns (see Fig. 5 (b)); in case (ii),  $T_t^{peak} = -114.8$  ns and  $T_t^{peak} = -138.0$  ns [see Fig. 5(d)]. From these data, we find that the peak time of the reflected pulse is changed much larger than that of the transmitted pulse. Comparing Figs. 5(a) with 5(c), we find that the reflectivity is increased by a large amount from 1.8% to 46.3% when the thickness of the slab is changed from the case (i) to the case (ii). Thus we can obtain large amount energy in the reflected pulse.

#### **C. The role of the slab**

In order to understand the role of the slab itself, we consider the slab without the doping atoms. For *d*  $=2m(\lambda_0/4\sqrt{\epsilon_b})$ , the slab itself is on resonance near the center frequency  $\omega_0$ ; and for  $d = (2m+1)(\lambda_0/4\sqrt{\epsilon_b})$  it is off resonance. Due to that the absorbing line (or gain line) of the doped atoms is extremely narrower than the resonant and off-resonant spectral width of the slab itself, the reflectivity (and transmittivity) is nearly independent of frequency under a fixed optical thickness and depends on the slab's optical thickness (see Fig. 6 for the reflectivity; it is nearly a constant under a fixed thickness; not shown for the transmittivity). For the optical thickness  $L = \sqrt{\epsilon_b}d = 2m(\lambda_0/4)$ , the reflection coefficient is almost equal to zero; under the offresonant case  $L = \sqrt{\epsilon_b}d = (2m+1)(\lambda_0/4)$ , the reflection coefficient is increased to another constant. In these two cases without the doping atoms, there are no superluminal phenomena at all. Comparing Fig. 2(a) with Fig. 6, we find that the reflectivity of the doped slab has a large change for different slab thickness; while the transmittivity has only a slight change. When the slab itself satisfies the resonant condition [i.e.,  $L = 2m(\lambda_0/4)$ ], the resonant condition will be broken because of the atoms doping into the slab. Thus the reflectivity near the absorbing (or gain) line of the doped atoms will be increased [see Fig. 1(a), 3(a), 4(a), and 5(a)]. When the slab itself satisfies the off-resonant condition [i.e.,  $L = (2m+1)(\lambda_0/4)$ , the slab itself dominates in the frequency

region out of the absorbing (or gain) line of the doped atoms; near or within the frequency region of the absorbing (or gain) line of the doped atoms, the property of the doped atoms dominates the reflection and transmission.

## **IV. CONCLUSIONS**

We have investigated the propagation properties of a light pulse incident into the slab systems doped with two-level atoms or three-level atoms. The doped atoms can be passive (absorptive) or active (gain). Our numerical results show that, with doped absorptive two-level atoms, the transmitted pulse is always superluminal [28], but the reflected pulse can be subluminal or superluminal depending on the thickness of the slab. When the slab's thickness  $d = (2m+1)(\lambda_0/4\sqrt{\epsilon_b})$ , both the reflected and transmitted pulses are superluminal. For the gain slab doped with the inverted two-level atoms, both the reflected and transmitted pulse are always subluminal; no superluminal phenomena occur even the thickness of the slab is changed. For the slab doped with absorptive threelevel atoms, the transmitted pulse is always subluminal; while for the reflected pulse, it is dependent on the condition of the slab: when the thickness *d* is equal to  $2m(\lambda_0/4\sqrt{\epsilon_b})$ , the reflected pulse is superluminal; when the thickness of the slab is equal to  $(2m+1)(\lambda_0/4\sqrt{\epsilon_b})$ , the reflected pulse is subluminal. For the gain slab doped with inverted three-level atoms, both the reflected and transmitted pulses are always superluminal. It should be pointed out that Manga Rao *et al.* [17] only consider the case of the resonance of the cavity (or slab) (i.e.,  $\omega_{cavity} = \omega_0$ , where  $\omega_{cavity}$  is the resonant frequency of the cavity). Our mechanism to control the pulse propagation from subluminal to superluminal in slab systems is different from Ref. [17]. By changing the thickness of the slab [i.e., changing the resonant condition of the slab (or the cavity)], we can control the pulse reflection from the superluminal to subluminal or vice versa.

In this paper, we present a simple method to achieve superluminal reflection and superluminal transmission simultaneously in the slab systems doped with the absorbing twolevel atoms and doped with the inverted three-level atoms. The reflected pulse can be controlled from superluminal to subluminal in the slab doped with the absorbing two-level atoms or the absorbing three-level atoms, by changing the thickness or the background dielectric constant (i.e., the background refraction index) of the slab.

#### **ACKNOWLEDGMENTS**

This work was supported by RGC and CA02/03.SC01 from the Government of Hong Kong, and FRG from Hong Kong Baptist University.

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